

The ζ - ζ Correlator Is Time Dependent

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We comment on the recent arguments by Senatore and Zaldarriaga that loop corrections to the ζ - ζ correlator cannot grow with time after first horizon crossing. We first emphasize the need to search for such secular dependence in corrections whose in-out matrix elements are infrared singular on an infinite spatial manifold. Then we give examples of such time dependence from pure quantum gravity and from scalar potential models. Finally, we point out that this time dependence arises from inflationary particle production and is therefore unlikely to endanger the preservation of super-horizon correlations as a record of inflation.

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INTRODUCTION

Perhaps the most commonly quoted result for models of inflation is the curvature power spectrum [1],

$$\Delta_{\mathcal{R}}^2(k, t) \equiv \frac{k^3}{2\pi^2} \int d^3x e^{-i\vec{k}\cdot\vec{x}} \left\langle \Omega \left| \mathcal{R}(t, \vec{x}) \mathcal{R}(t, \vec{0}) \right| \Omega \right\rangle. \quad (1)$$

(The field \mathcal{R} is defined by stripping the derivatives from the 3-curvature in the co-moving frame for which the momentum flux vanishes [1].) These predictions are made in the context of perturbation theory about a homogeneous, isotropic and spatially flat geometry,

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \Rightarrow H \equiv \frac{\dot{a}}{a}, \quad \epsilon \equiv -\frac{\dot{H}}{H^2}. \quad (2)$$

The time of first horizon crossing is t_k such that $k = H(t_k)a(t_k)$, after which $\Delta_{\mathcal{R}}^2(k, t)$ becomes nearly constant and one drops the argument t . The tree order result for typical single-scalar inflation models is [2],

$$\Delta_{\mathcal{R}}^2(k) \approx \frac{GH^2(t_k)}{\pi\epsilon(t_k)}. \quad (3)$$

Theorists are eager to predict $\Delta_{\mathcal{R}}^2(k)$ because its value for cosmological wave lengths can be reconstructed from observations of anisotropies in the cosmic microwave radiation and from large scale structure surveys [3],

$$\Delta_{\mathcal{R}}^2(k) = \left(2.441_{-0.092}^{+0.088}\right) \times 10^{-9} \left(\frac{k}{.002 \text{ Mpc}^{-1}}\right)^{-0.037 \pm 0.012} \quad (4)$$

This connection between quantum gravity and cosmological observation represents one of the great triumphs of inflation theory [4], and accords expression (4) the status of the first quantum gravitational data ever obtained.

Tree order results such as (3) derive from the linearized mode functions. There can be important contributions from times before t_k [5], but the mode functions become constant afterwards because the restoring force $k^2/a^2(t)$ redshifts away while the friction term remains

large. Quantum loop effects can induce late time dependence by coupling mode k to changes in the vacuum energy from the quantum fluctuations of other modes. A theorem by Weinberg limits this time dependence to powers of the “infrared logarithm”, $\ln[a(t)/a(t_k)]$ [6–8]. No one disputes this bound, the issue is its saturation.

In his first paper on the subject Weinberg considered two one loop processes which seemed to contribute infrared logarithms [6]:

- Section V gave a qualitative treatment of self-interactions within the gravity-inflaton system, culminating in equation (41); and
- Section VII gave a computation of the contribution from \mathcal{N} free, massless, minimally coupled scalars, culminating in equation (71).

Although other work has produced similar results [9, 10], Senatore and Zaldarriaga have argued that there cannot be any infrared logarithms from Weinberg’s second source [11]. We agree — indeed, this follows from a simple rule for counting infrared logarithms [12]. However, we do not accept the subsequent conclusion by Senatore and Zaldarriaga that $\Delta_{\mathcal{R}}^2(k, t)$ is free of infrared logarithms from any source and to all orders. (A recent paper by Giddings and Sloth also disputes their conclusion [13].) The purpose of this paper is to show that infrared logarithms arise at one loop from self-interactions of the gravity-inflaton system (as in Weinberg’s first example) and at two loops from massless, minimally coupled scalars with a quartic potential.

In section 2 we summarize the Lagrangian. Section 3 describes a simple rule for counting the maximum number of infrared logarithms which can derive from a given interaction [12]. In section 4 we compute a one loop effect from self-interactions of the gravity-inflaton system. Section 5 gives a two loop effect from the potential of a massless, minimally coupled scalar. Our conclusions comprise the final section.

GAUGE-FIXED LAGRANGIAN

The model we consider consists of three fields: the spacelike, D -dimensional metric $\mathbf{g}_{\mu\nu}$; the scalar inflaton φ whose slow roll down its potential $V(\varphi)$ drives inflation; and a spectator scalar σ which is centered at the $\sigma_0 = 0$ minimum (of zero) of its massless potential $U(\sigma)$. The Lagrangian is,

$$\mathcal{L} = \left[\frac{\mathbf{R}}{16\pi G} - \frac{1}{2} \varphi_{,\mu} \varphi_{,\nu} \mathbf{g}^{\mu\nu} - V(\varphi) - \frac{1}{2} \sigma_{,\mu} \sigma_{,\nu} \mathbf{g}^{\mu\nu} - U(\sigma) \right] \sqrt{-\mathbf{g}}, \quad (5)$$

where \mathbf{R} is the D -dimensional Ricci scalar and a comma denotes ordinary differentiation.

We decompose $\mathbf{g}_{\mu\nu}$ into lapse, shift and spatial metric according to Arnowitt, Deser and Misner (ADM) [14],

$$\mathbf{g}_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + g_{ij} (dx^i - N^i dt)(dx^j - N^j dt). \quad (6)$$

ADM long ago showed that the Lagrangian has a very simple dependence upon the lapse [14],

$$\mathcal{L} = \left(\text{Surface Terms} \right) - \frac{\sqrt{g}}{16\pi G} \left[N \cdot A + \frac{B}{N} \right]. \quad (7)$$

The quantity A is a potential energy,

$$A = -R + 16\pi G \left[V(\varphi) + U(\sigma) + \frac{1}{2} g^{ij} \left(\varphi_{,i} \varphi_{,j} + \sigma_{,i} \sigma_{,j} \right) \right], \quad (8)$$

where R is the $(D-1)$ -dimensional Ricci scalar formed from g_{ij} . The quantity B in (7) is a sort of kinetic energy,

$$B = (E^i_i)^2 - E^{ij} E_{ij} - 8\pi G \left[\left(\dot{\varphi} - \varphi_{,i} N^i \right)^2 + \left(\dot{\sigma} - \sigma_{,i} N^i \right)^2 \right], \quad (9)$$

where $E_{ij}/2N$ is the extrinsic curvature,

$$E_{ij} \equiv \frac{1}{2} \left[N_{i;j} + N_{j;i} - \dot{g}_{ij} \right], \quad (10)$$

and a semi-colon denotes covariant differentiation. Varying (7) with respect to N produces an algebraic equation,

$$A - \frac{B}{N^2} = 0 \quad \implies \quad N = \sqrt{\frac{B}{A}} \quad (11)$$

This gives the constrained Lagrangian a ‘‘virial’’ form,

$$\mathcal{L}_{\text{const}} = \left(\text{Surface Terms} \right) - \frac{\sqrt{g}}{8\pi G} \sqrt{AB}. \quad (12)$$

Further progress requires the use of perturbation theory. The nonzero background fields are $g_{ij} = a^2(t) \delta_{ij}$ and $\varphi = \varphi_0(t)$. The two nontrivial Einstein equations can be used to eliminate the background scalar,

$$\dot{\varphi}_0^2 = -\frac{(D-2)}{8\pi G} \dot{H}, \quad V(\varphi_0) = \frac{(D-2)}{16\pi G} \left[\dot{H} + (D-1)H^2 \right]. \quad (13)$$

Note that the background values of the potential and kinetic terms are equal, $A_0 = B_0 = (D-2)[\dot{H} + (D-1)H^2]$. Hence the background value of the lapse is unity.

We fix time as Maldacena [15] and Weinberg [6],

$$G_0(t, \vec{x}) \equiv \varphi(t, \vec{x}) - \varphi_0(t) = 0. \quad (14)$$

The other $(D-1)$ conditions have to do with how we define the unimodular part of the metric \tilde{g}_{ij} ,

$$g_{ij} = a^2(t) e^{2\zeta(t, \vec{x})} \tilde{g}_{ij}(t, \vec{x}) \implies \sqrt{g} = a^{D-1} e^{(D-1)\zeta}. \quad (15)$$

We require $\tilde{g}_{ij} \equiv \delta_{ij} + h_{ij}$ to be transverse,

$$G_i(t, \vec{x}) \equiv \partial_j \tilde{g}_{ij}(t, \vec{x}) = \partial_j h_{ij}(t, \vec{x}) = 0. \quad (16)$$

(Maldacena and Weinberg imposed transversality on the logarithm of \tilde{g}_{ij} .) The resulting Faddeev-Popov determinant depends only on h_{ij} , and is singular for $\epsilon = 0$.

Of course no gauge can eliminate physical inflatons; with condition (14) that degree of freedom resides in $\zeta(t, \vec{x})$. Linearized gravitons are carried by $h_{ij}(t, \vec{x})$, and spectator scalars are in $\sigma(t, \vec{x})$. By contrast, the shift field $N^i(t, \vec{x})$ is a constrained variable which mediates interactions between the other fields.

To reach a perturbative form we first employ (15) to exhibit how the potential (8) depends on ζ , h_{ij} and σ ,

$$A = A_0 - R + 16\pi G \left[U(\sigma) + \frac{e^{-2\zeta}}{2a^2} \tilde{g}^{ij} \sigma_{,i} \sigma_{,j} \right] \equiv A_0(1 + \alpha). \quad (17)$$

Here the spatial Ricci scalar is,

$$R = \frac{e^{-2\zeta}}{a^2} \left[\tilde{R} - 2(D-2)\tilde{\nabla}^2 \zeta - (D-2)(D-3)\zeta^{,k} \zeta_{,k} \right], \quad (18)$$

where $\tilde{R} = O(h^2)$ is the Ricci scalar formed from \tilde{g}_{ij} and $\tilde{\nabla}^2 \equiv \partial_i \tilde{g}^{ij} \partial_j$ is the covariant scalar Laplacian. At this stage we can also recognize that \mathcal{R} is just ζ , in $D = 4$ dimensions and to linearized order [1],

$$\mathcal{R}(t, \vec{x}) \equiv -\frac{a^2(t)}{4\nabla^2} R = \left(\frac{D-2}{2} \right) \zeta(t, \vec{x}) + O(\zeta^2, \zeta h, h^2). \quad (19)$$

The kinetic energy (9) can be expressed as,

$$\begin{aligned} B &= A_0 + 2(D-2)H \left[(D-1)(\dot{\zeta} - \zeta_{,k} \tilde{N}^k) - \tilde{N}^k_{,k} \right] \\ &\quad + (D-2)(\dot{\zeta} - \zeta_{,k} \tilde{N}^k) \left[(D-1)(\dot{\zeta} - \zeta_{,k} \tilde{N}^k) - 2\tilde{N}^k_{,k} \right] \\ &\quad + (\tilde{N}^k_{,k})^2 - \tilde{E}^{k\ell} \tilde{E}_{k\ell} - 8\pi G (\dot{\sigma} - \sigma_{,k} \tilde{N}^k)^2, \quad (20) \\ &\equiv A_0(1 + \beta). \quad (21) \end{aligned}$$

Here we define $\tilde{N} \equiv N^i$, $\tilde{N}_i \equiv \tilde{g}_{ij} \tilde{N}^j$ and $\tilde{E}_{ij} \equiv \frac{1}{2} [\tilde{N}_{i;j} + \tilde{N}_{j;i} - \dot{\tilde{g}}_{ij}]$. The next step is to expand the volume part of the constrained Lagrangian in powers of α and β ,

$$-\frac{\sqrt{g}}{8\pi G} \sqrt{AB} = -\frac{a^{D-1} e^{(D-1)\zeta}}{8\pi G} A_0 \sqrt{(1+\alpha)(1+\beta)} \quad (22)$$

$$= -\frac{a^{D-1} e^{(D-1)\zeta}}{8\pi G} A_0 \left\{ 1 + \frac{(\alpha+\beta)}{2} - \frac{(\alpha-\beta)^2}{8} + \dots \right\}. \quad (23)$$

As Weinberg noted, the terms involving no derivatives of the gravity fields sum up to a total derivative [6]. Another important fact is that quadratic mixing between \tilde{N}^i and ζ can be eliminated with the covariant field redefinition,

$$\tilde{S}^k \equiv \tilde{N}^k + \tilde{g}^{k\ell} \partial_\ell \frac{1}{\nabla^2} \left[\frac{e^{-2\zeta}}{Ha^2} \tilde{\nabla}^2 \zeta - \epsilon (\dot{\zeta} - \zeta_{,i} \tilde{N}^i) \right]. \quad (24)$$

After much work the quadratic Lagrangians emerge,

$$\mathcal{L}_S^{(2)} = \frac{a^{D-1}}{32\pi G} \left\{ \partial_\ell \tilde{S}^k \partial_\ell \tilde{S}^k + \left(\frac{D-3+\epsilon}{D-1-\epsilon} \right) \partial_\ell \tilde{S}^k \partial_k \tilde{S}^\ell \right\}, \quad (25)$$

$$\mathcal{L}_\zeta^{(2)} = \frac{(D-2)\epsilon a^{D-1}}{16\pi G} \left\{ \dot{\zeta}^2 - \frac{1}{a^2} \partial_k \zeta \partial_k \zeta \right\}, \quad (26)$$

$$\mathcal{L}_h^{(2)} = \frac{a^{D-1}}{64\pi G} \left\{ \dot{h}_{ij} \dot{h}_{ij} - \frac{1}{a^2} \partial_k h_{ij} \partial_k h_{ij} \right\}, \quad (27)$$

$$\mathcal{L}_\sigma^{(2)} = \frac{a^{D-1}}{2} \left\{ \dot{\sigma}^2 - \frac{1}{a^2} \partial_k \sigma \partial_k \sigma \right\}. \quad (28)$$

Expression (28) reveals σ to be a massless, minimally coupled scalar with unit normalization. Let us call its propagator $i\Delta(x; x')$. From (27), and relations (15-16), we see that the graviton propagator is proportional,

$$i[\Delta_{k\ell}](x; x') = 32\pi G \left[\Pi_{i(k} \Pi_{\ell)j} - \frac{\Pi_{ij} \Pi_{k\ell}}{D-2} \right] i\Delta(x; x'), \quad (29)$$

where $\Pi_{ij} \equiv \delta_{ij} - \partial_i \partial_j / \nabla^2$ is the transverse projection operator. These relations are exact. Because $\epsilon(t) = -\dot{H}/H^2$ is nearly constant during inflation, expression (26) implies a similar relation for the ζ propagator,

$$i\Delta_\zeta(x; x') \approx \frac{8\pi G}{(D-2)\epsilon} i\Delta(x; x'). \quad (30)$$

The massless, minimally coupled scalar has a well-known infrared problem [16] which we regulate by working on T^{D-1} with radius L and then making the integral approximation for the mode sum [17],

$$i\Delta(x; x') = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \theta(k - L^{-1}) e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} \times \left\{ \theta(t - t') u(t, k) u^*(t', k) + \theta(t' - t) u^*(t, k) u(t', k) \right\}. \quad (31)$$

The mode function for constant ϵ is,

$$u(t, k) = \frac{\sqrt{\frac{\pi}{4(1-\epsilon)H}}}{a^{\frac{D-1}{2}}} H_\nu^{(1)} \left(\frac{k}{(1-\epsilon)Ha} \right), \quad \nu \equiv \frac{D-1-\epsilon}{2(1-\epsilon)}. \quad (32)$$

Constant ϵ implies Ha^ϵ is also constant and hence,

$$D-4=0=\dot{\epsilon} \implies \lim_{t \rightarrow \infty} u(t, k) = C(\epsilon) \times \frac{H(t_k)}{\sqrt{2k^3}}, \quad (33)$$

where $C(0) = 1$ and we will use $C(\epsilon) \approx 1$ generally.

Relations (19), (30) and (33) allow a trivial derivation of the typical result (3) for the scalar power spectrum,

$$\left[\Delta_{\mathcal{R}}^2(k, t) \right]_{\text{tree}} \approx \frac{k^3}{2\pi^2} \times \frac{8\pi G}{2\epsilon} \times |u(t, k)|^2 \approx \frac{GH^2(t_k)}{\pi\epsilon}. \quad (34)$$

The tensor power spectrum is,

$$\Delta_h^2(k, t) \equiv \frac{k^3}{2\pi^2} \int d^3x e^{-i\vec{k} \cdot \vec{x}} \left\langle \Omega \left| h_{ij}(t, \vec{x}) h_{ij}(t, \vec{0}) \right| \Omega \right\rangle. \quad (35)$$

Relations (29) and (33) offer a similarly straightforward derivation of the typical tree result for $\Delta_h^2(k, t)$ [1, 2],

$$\left[\Delta_h^2(k, t) \right]_{\text{tree}} = \frac{k^3}{2\pi^2} \times 32\pi G \times 2 \times |u(t, k)|^2 \approx \frac{16}{\pi} GH^2(t_k). \quad (36)$$

The $1/\epsilon$ enhancement of the scalar power spectrum with regard to the tensor one presumably explains why the scalar contribution has been detected but the tensor signal has so far not been resolved. At 95% confidence the bound on their ratio at $k_0 = 0.002 \text{ Mpc}^{-1}$ is [3],

$$r \equiv \frac{\Delta_h^2(k_0)}{\Delta_{\mathcal{R}}^2(k_0)} < 0.22. \quad (37)$$

With the typical tree order results (3) and (36), and the measured spectrum (4), this bound implies an upper limit on the inflationary Hubble parameter,

$$GH^2(t_{k_0}) \approx \frac{\pi}{16} \times r \times \Delta_{\mathcal{R}}^2(k_0) \lesssim 10^{-10}. \quad (38)$$

We can also get a bound on ϵ by combining the typical tree results (3) and (36) with (37),

$$\epsilon(t_{k_0}) \approx \frac{r}{16} \lesssim 0.014. \quad (39)$$

This is why it was justified to use $C(\epsilon) \approx 1$. *We shall go further and approximate loop corrections by taking the de Sitter limit of $\epsilon = 0$ once multiplicative factors of ϵ have been removed from vertices and propagators.*

It remains to derive the relevant interactions by expanding the constrained Lagrangian (23). These interactions are quite complicated, but most of them are precluded by too many differentiated fields from contributing the maximum number of infrared logarithms. If we want just the maximum possible number of infrared logarithms then the number of interactions at any order becomes manageable. To study the lowest order effects of ζ self-interactions it suffices to consider the minimal generalization of (26),

$$\mathcal{L}_\zeta = \frac{(D-2)\epsilon}{16\pi G} a^{D-1} e^{(D-1)\zeta} \left\{ \dot{\zeta}^2 - \frac{e^{-2\zeta}}{a^2} \partial_k \zeta \partial_k \zeta \right\}. \quad (40)$$

To study the lowest order effects of the scalar potential we need only,

$$\mathcal{L}_U = \frac{\epsilon}{D-1} a^{D-1} e^{(D-1)\zeta} U(\sigma). \quad (41)$$

INFRARED LOGARITHMS

Infrared logarithms are factors of $\ln[a(t)]$ which can contaminate loop corrections involving undifferentiated

gravitons or massless, minimally coupled scalars. The oldest example is from 1982 [18] and consists of the coincidence limit of $i\Delta(x; x')$ on de Sitter background ($\epsilon(t) = 0$ and $H(t) = H_I$). With full dimensional regularization, and $L = (H_I a_I)^{-1}$, the result is [17],

$$i\Delta(x; x) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \theta(k - H_I a_I) |u(t, k)|^2, \quad (42)$$

$$= \frac{a^{-(D-1)}}{2^D \pi^{\frac{D-3}{2}} \Gamma(\frac{D-1}{2})} \int_{H_I a_I}^{\infty} dk k^{D-2} \left| H_{\frac{D-1}{2}}^{(1)}\left(\frac{k}{H_I a}\right) \right|^2, \quad (43)$$

$$= \frac{H_I^{D-2}}{2^D \pi^{\frac{D-3}{2}} \Gamma(\frac{D-1}{2})} \int_{a_I a^{-1}}^{\infty} dz z^{D-2} \left| H_{\frac{D-1}{2}}^{(1)}(z) \right|^2, \quad (44)$$

$$= \frac{H_I^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ 2 \ln \left[\frac{a(t)}{a_I} \right] - \psi \left(1 - \frac{D}{2} \right) + \psi \left(\frac{D-1}{2} \right) + \psi(D-1) + \psi(1) + O\left(\frac{a_I^2}{a^2}\right) \right\}. \quad (45)$$

This exhibits the fallacy of the argument Senatore and Zaldarriaga gave against infrared logarithms based on “making the integral dimensionless” [11]. That is the change of variables from k to $z = k/H_I a(t)$ in passing from (43) to (44). Had the lower limit been $k = 0$ this would indeed have eliminated any time dependence, however, the integral would have been infrared divergent. So we come to a crucial insight: *infrared logarithms derive from diagrams that would be infrared divergent as in-out matrix elements on the spatial manifold R^{D-1}* . Of course that is why they are called infrared logarithms.

Note that any derivatives, with respect to space or time, would have eliminated the infrared logarithm in (45). This observation has led to a very simple rule for inferring the maximum number of infrared logarithms which can come from a particular interaction [12]: *If the interaction has a total of K undifferentiated gravitons and undifferentiated massless, minimally coupled scalars, after partial integration has been exhausted, then each correction involving two such interactions can produce as many as K infrared logarithms.*

This rule has been tested in a variety of explicit, fully dimensionally regulated and renormalized computations on de Sitter background using the Schwinger-Keldysh formalism [19]. For a massless, minimally coupled scalar with a quartic self-interaction the rule correctly predicts the number of infrared logarithms in the expectation value of the stress tensor at one and two loop orders [20], and in the one and two loop order self-mass-squared [21, 22]. For scalar quantum electrodynamics the rule correctly predicts the infrared logarithms which are seen in the one loop vacuum polarization [23] and in the two loop scalar and electrodynamic field strengths, as well as the two loop expectation value of the stress tensor [24]. For a Yukawa-coupled scalar the rule has been checked with the one loop fermion self-energy [25] and with the expectation value of the coincident vertex at two loop order [26]. The rule also gives the correct number of

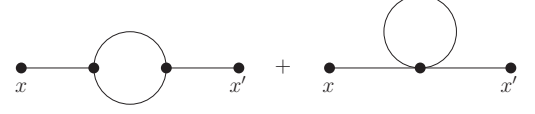


FIG. 1: One loop correction from cubic and quartic self-interactions of ζ .

infrared logarithms in the one loop fermion self-energy from quantum gravity [27].

Senatore and Zaldarriaga considered two models in detail. The first, given in their equation (11) and studied in section 3, consists of a Lagrangian containing only differentiated fields [11]. Any such interaction has $K = 0$, so the rule predicts no infrared logarithms, which is what they found. Their second model, given in their equation (75) and studied in section 4, was the same as Weinberg’s: gravity + inflaton + N massless, minimally coupled scalars with no potential [11]. This model shows infrared logarithms, both from its ζ self-interactions and from interactions with undifferentiated h_{ij} fields. However, Senatore and Zaldarriaga ignored those interactions because they give no parametric enhancement involving the potentially large number N . The scalar kinetic terms which were the object of their study consist of differentiated σ fields with a complicated set of couplings to one ζ field. Although there are certainly some $K = 1$ terms present, cancellations make the resulting integrals infrared finite [6, 11] so the rule again predicts no infrared logarithms, and that is what they found.

Let us now consider the two interactions (40) and (41) given in the previous section. The general form of the ζ self-interaction (40) is $\zeta^K \partial \zeta \partial \zeta$, which is the same as for quantum gravity. We therefore expect that there should be a single infrared logarithm from a correction involving two 3-point interactions, with $K = 1$, or from a single 4-point interaction, with $K = 2$. These corrections correspond to the diagrams depicted in Fig. 1, and we will show in the next section that they indeed produce a single infrared logarithm. Supposing that the spectator potential is quartic, we see that (41) contains an interaction of the form $\zeta^2 \sigma^4$. This has $K = 6$, so the rule predicts three infrared logarithms from a correction which involves one such interaction. The corresponding diagram is depicted in Fig. 2 and we will confirm that it does produce three infrared logarithms in the penultimate section.

TIME DEP. FROM SELF-INTERACTIONS OF ζ

The two diagrams of Fig. 1 derive from expanding expression (40) to cubic and quartic orders. The second of these diagrams is very similar to the computation featured in section V of Weinberg’s paper [6]. As he noted, a field redefinition would make (40) free were it not for the

extra factor of $e^{-2\zeta}$ on the term with space derivatives. Things can be simplified by exploiting Weinberg's observation that only the time derivative term contributes an infrared logarithm at one loop order [6]. We therefore make the field redefinition,

$$Z \equiv \frac{2}{D-1} \left[e^{\frac{D-1}{2}\zeta} - 1 \right] \Leftrightarrow \zeta = \frac{2}{D-1} \ln \left[1 + \frac{D-1}{2} Z \right], \quad (46)$$

and forget about the residual interactions involving spatial derivatives.

The Z propagator is the same as the ζ propagator (30). Hence the one ζ loop correction to the ζ - ζ correlator is,

$$\begin{aligned} \langle \Omega | \zeta(x) \zeta(x') | \Omega \rangle_{\zeta \text{ loop}} &\approx \left(\frac{D-1}{2} \right)^2 \langle \Omega | \frac{1}{3} Z(x) Z^3(x') \\ &\quad + \frac{1}{4} Z^2(x) Z^2(x') + \frac{1}{3} Z^3(x) Z(x') | \Omega \rangle, \quad (47) \\ &\approx \left(\frac{D-1}{2} \right)^2 \left[\frac{8\pi G}{(D-2)\epsilon} \right]^2 \left\{ i\Delta(x; x') i\Delta(x'; x') \right. \\ &\quad \left. + \frac{1}{2} [i\Delta(x; x')]^2 + i\Delta(x; x) i\Delta(x; x') \right\}. \quad (48) \end{aligned}$$

We now set $x^\mu = (t, \vec{x})$ and $x'^\mu = (t, \vec{0})$, and Fourier transform on \vec{x} . It also makes sense to retain only the infrared logarithm terms because time independent contributions derive as well from derivative interactions of the same order as (40) which we have ignored. That is where the ultraviolet divergences reside, and they can be absorbed into BPHZ counterterms as usual. In the absence of any condition for fixing the finite parts of those counterterms, the infrared logarithm terms are the only unambiguous prediction. The final result is,

$$\left[\Delta_{\mathcal{R}}^2(k, t) \right]_{\zeta \text{ loops}} \approx \frac{GH^2}{\pi\epsilon} \left\{ \frac{27GH^2}{4\pi\epsilon} \ln(a) + O(G^2 H^4) \right\}. \quad (49)$$

We should mention that it is by no means clear what collection of fields represents the observed scalar power spectrum $\Delta_{\mathcal{R}}^2(k, t)$. At tree order it suffices to use the ζ - ζ correlator, and our result (49) is based on extending that correspondence to all orders. This definition affords a simple renormalization scheme because then the power spectrum is a noncoincident Green's function of a fundamental field, and ordinary renormalization makes those finite. However, it is conceivable that the measured quantity is actually the correlator of some composite operator such as (19), in which case an additional, composite operator renormalization would be required. Nonlinear modifications of the observable can introduce additional infrared logarithms. For example, if the correct observable is the correlator of the field $Z(t, \vec{x})$ defined in expression (46), then there are no infrared logarithms at one loop order. However, there does not seem any reason to suppose this, and even doing so would not prevent the appearance of infrared logarithms at higher orders.

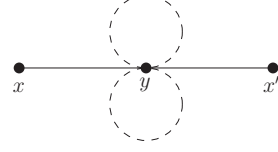


FIG. 2: Two loop correction from the interaction given in expression (41). Solid lines represent ζ and dashed lines represent σ .

TIME DEP. FROM SPECTATOR POTENTIALS

Let us assume $U(\sigma) = \lambda\sigma^4/4!$. The diagram of Fig. 2 derives from the $\zeta^2\sigma^4$ term of the interaction (41),

$$\Delta\mathcal{L} = \frac{(D-1)}{48} \lambda \epsilon a^{D-1} \zeta^2 \sigma^4. \quad (50)$$

The Schwinger-Keldysh [19] result for this diagram is,

$$\begin{aligned} (\text{Fig. 2}) &\approx \left[\frac{8\pi G}{(D-2)\epsilon} \right]^2 \int d^D y \left\{ i\Delta_{++}(x; y) i\Delta_{++}(x'; y) \right. \\ &\quad \left. - i\Delta_{+-}(x; y) i\Delta_{+-}(x'; y) \right\} \frac{i\lambda(D-1)}{8} \epsilon a^{D-1} [i\Delta(y; y)]^2. \quad (51) \end{aligned}$$

The propagator $i\Delta_{++}(x; x')$ is the same mode sum as (31), whereas $i\Delta_{+-}(x; x')$ has the same first line as (31) but the curly-bracketed expression on the second line is replaced by just $u^*(t, k)u(t', k)$.

We again take $x^\mu = (t, \vec{x})$ and $x'^\mu = (t, \vec{0})$, and Fourier transform on \vec{x} , to obtain,

$$\begin{aligned} \int d^{D-1}x e^{-i\vec{k}\cdot\vec{x}} (\text{Fig. 2}) &= \frac{i8\pi^2(D-1)\lambda G^2}{(D-2)^2\epsilon} \int_0^t ds [a(s)]^{D-1} \\ &\quad \times \left\{ [u(t, k)]^2 [u^*(s, k)]^2 - [u^*(t, k)]^2 [u(s, k)]^2 \right\} [i\Delta]^2. \quad (52) \end{aligned}$$

There is no point in retaining the divergent part of the coincident propagator (45), and continuing to work in D dimensions, unless we add the various counterterm diagrams. That exercise is identical to the published two loop computation of the expectation value of the σ stress tensor [20]. We will therefore retain only the leading infrared logarithm terms and take $D = 4$.

Oscillations of the mode functions preclude a coherent effect before first horizon crossing. After horizon crossing one may take the long wavelength limit of the mode functions,

$$u(t, k) \longrightarrow \frac{H}{\sqrt{2k^3}} \left\{ 1 + \frac{1}{2} \left(\frac{k}{Ha} \right)^2 + \frac{i}{3} \left(\frac{k}{Ha} \right)^3 + \dots \right\}. \quad (53)$$

Hence the curly-bracketed term of (52) becomes,

$$\begin{aligned} &\left\{ [u(t, k)]^2 [u^*(s, k)]^2 - [u^*(t, k)]^2 [u(s, k)]^2 \right\} \\ &\longrightarrow -\frac{iH}{k^3} \left\{ \frac{1}{3} \left[\frac{1}{a^3(s)} - \frac{1}{a^3(t)} \right] + O\left(\frac{k^2}{H^2}\right) \right\}. \quad (54) \end{aligned}$$

Putting everything together produces,

$$\left[\int d^{D-1}x e^{-i\vec{k}\cdot\vec{x}} (\text{Fig. 2}) \right]_{\text{leading log}} \approx \frac{\lambda G^2 H^5}{8\pi^2 \epsilon k^3} \int_{t_k}^t ds \left[1 - \frac{a^3(s)}{a^3(t)} \right] \ln^2[a(s)], \quad (55)$$

$$= \frac{\lambda G^2 H^4}{24\pi^2 \epsilon k^3} \left\{ \ln^3[a(t)] + \text{subleading} \right\}. \quad (56)$$

And multiplying by $k^3/2\pi^2$ gives the power spectrum,

$$\left[\Delta_{\mathcal{R}}^2(k, t) \right]_{\sigma \text{ loops}} \approx \frac{GH^2}{\pi \epsilon} \left\{ \frac{\lambda GH^2}{48\pi^3} \ln^3(a) + O(\lambda^2) \right\}. \quad (57)$$

CONCLUSIONS

We have shown that the ζ - ζ correlator acquires time dependent infrared log corrections, starting at one loop (49) from ζ self-interactions, and at two loops (57) from the quartic potential of a spectator scalar. There should also be infrared logarithms from dynamical gravitons, starting at two loops.

The physical interpretation of the spectator effect (57) derives from the small increase in the vacuum energy as inflationary particle production pushes the σ field up its potential $U(\sigma)$. The dimensionally regulated and fully renormalized result for this has been derived [20], but we can understand its effect on the ζ mode functions by simply adding the Hartree approximation of (50) to the free ζ Lagrangian (26) in $D = 4$ dimensions,

$$\mathcal{L}_{\zeta}^{(2)} \longrightarrow \frac{\epsilon a^3}{8\pi G} \left\{ \dot{\zeta}^2 - \frac{1}{a^2} \partial_k \zeta \partial_k \zeta + \frac{3\lambda GH^4}{32\pi^3} \ln^2(a) \zeta^2 \right\}. \quad (58)$$

After horizon crossing the associated mode equation is,

$$3H\dot{u} \approx \frac{3\lambda GH^4}{32\pi^3} \ln^2(a) u \\ \implies u(t, k) \approx \frac{H}{\sqrt{2k^3}} \left\{ 1 + \frac{\lambda GH^2}{96\pi^3} \ln^3(a) \right\}. \quad (59)$$

Inserting the quantum corrected mode function in expression (34) gives precisely our result (57). It seems likely that a similar explanation can be given for the effects from ζ self-interactions, and from interactions with gravitons.

Such effects must be present or else there is something seriously wrong with our understanding of how gravity responds to quantum fluctuations. That they would even be questioned is a tribute to how firmly cosmologists have come to believe in the time independence of $\tilde{\zeta}(t, \vec{k})$ after horizon crossing. Of course we appreciate the wonder of preserving a memory of conditions from inflation, but the practical value of $\Delta_{\mathcal{R}}^2(k, t)$ does not seem compromised by the minuscule time dependence we have exhibited. The loop corrections we have discussed can never be

large (which is the same conclusion reached by Weinberg [7, 8]) because they are suppressed by the quantum gravitational loop counting parameter $GH^2 \lesssim 10^{-10}$. Their enhancement by $\ln[a(t)/a(t_k)] \lesssim 60$ is huge by the standards of conventional perturbation theory, and unprecedented in view of its time dependence, but there are simply not enough e-foldings of inflation left after first horizon crossing to overcome the suppression factor for any mode whose spatial variation we can now perceive.

Despite having reached a different conclusion from Senatore and Zaldarriaga, our results represent no real disagreement with their analysis. They were uninterested in self-interactions from the gravity-inflaton system because the fixed number of fields in that sector cannot engender effects which are enhanced by a potentially large parameter such as Weinberg's \mathcal{N} . And they dismissed massless scalars with nonzero potentials as unnatural. We feel it is not reasonable to fine tune the inflaton potential $V(\varphi)$ and then quibble about fine tuning the spectator potential $U(\sigma)$. We also thought it worth establishing that infrared logarithms do contaminate gauge invariant quantum gravity observables such as $\Delta_{\mathcal{R}}^2(k, t)$ because the contrary view has been expressed [28, 29].

We close with two thoughts. First, the small infrared log corrections to $\Delta_{\mathcal{R}}^2(k, t)$ might eventually be observable through 21 centimeter measurements of the matter power spectrum out to very large redshifts [30]. This would require untangling the primordial signal from late time effects, which is very hard but perhaps not impossible. It would also require a precise tree order prediction from some unique model of inflation.

Our final comment is that loop corrections to the power spectrum are not the best place to study infrared logarithms because $\ln[a(t)/a(t_k)]$ cannot exceed about 60 for any mode whose spatial variation we now perceive. By contrast, there can be spectacular enhancements in quantities which seem spatially constant, such as the vacuum energy [31] and Newton's constant [32], because they receive contributions from modes which are still super-horizon. For a very long period of inflation perturbation theory can even break down, after which reliable computations would require some nonperturbative resummation technique. Such a method has been devised by Starobinsky [33], and applied by him and Yokoyama to scalar potential models [34], for which it sums the series of leading infrared logarithms [12]. Starobinsky's method has recently been extended to Yukawa-coupled fermions [26] and to scalar quantum electrodynamics [35]. It has not yet been extended to quantum gravity but there are reasons for believing that some version of it can be [36].

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